

Magnetic Phases in Dense Quark Matter

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Abstract. In this paper I discuss the magnetic phases of the three-flavor color superconductor. These phases can take place at different field strengths in a highly dense quark system. Given that the best natural candidates for the realization of color superconductivity are the extremely dense cores of neutron stars, which typically have very large magnetic fields, the magnetic phases here discussed could have implications for the physics of these compact objects.

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INTRODUCTION

One of the fundamental tasks of modern nuclear physics is to understand the phase structure of QCD. The study of the phase structure of QCD requires exploring regions of different baryon densities and temperatures. As known, at very high temperatures QCD should be in a deconfined phase: the quark-gluon plasma. This phase existed in the early universe and might be earthly reproduced in high energy experiments. A deconfined phase can also exist in a very different region of the temperature/density plane, that of low temperatures and very high densities. This region is often called dense QCD, although it really means cold and dense QCD. In this environment the baryons get so squeezed that they start to overlap, erasing any vestige of structure and liberating the quarks inside them.

The ground state of a superdense quark system, a Fermi liquid of weakly interacting quarks, is unstable with respect to the formation of diquark condensates, a non-perturbative phenomenon essentially equivalent to the Cooper instability of BCS superconductivity. In QCD, one gluon exchange between two quarks is attractive in the color-antitriplet channel. Thus, at sufficiently high density and sufficiently small temperature, quarks should condense into Cooper pairs, which are color antitriplets. These color condensates break the SU(3) color gauge symmetry of the ground state producing a color superconductor (CS) [1].

The most likely candidates for the realization of the phenomenon of color superconductivity are the core of compact stars, whose density can reach values several times larger than the saturation density of nuclear matter. Besides being very dense objects, compact stars typically have very large magnetic fields too. In the case of magnetars [2], fields in the range of $10^{14} - 10^{16}$ G have been predicted and observed. The quest to find observable signatures of a color-superconducting core must not ignore the presence of the star's magnetic field and its effects in the superconducting state.

Although a color superconductor is in principle an electric superconductor, because the diquark condensate carries nonzero electric charge, in the color-flavor-locked (CFL)

phase [3] (CFL is the color-superconducting phase that is realized in a system of three massless flavors at high densities) there is no Meissner effect for a new in-medium electromagnetic field \tilde{A}_μ . This in-medium (also called "rotated") electromagnetic field is a combination of the regular electromagnetic field and the 8th gluon [3, 4]. As the quark pairs are all neutral with respect to the "rotated" electromagnetic charge \tilde{Q} , the "rotated" electromagnetic field \tilde{A}_μ remains long-range within the superconductor.

Because of the lack of a Meissner effect in the color superconductor, the last one can be penetrated by the rotated component of an external magnetic field. Depending on its strength, an external magnetic field can then produce two different phases on a three-flavor color superconductor: the Magnetic-Color-Flavor-Locked (MCFL) phase [5] and the Paramagnetic CFL (PCFL) one [6, 7]. The properties and relevance of each of them at different magnetic scales have been investigated in Ref. [8]. The present paper summarizes the main characteristics of these phases, as well as the region of field strengths and chemical potentials where they (and the CFL) can occur, and the type of transition that takes from one to another.

CFL-MCFL TRANSMUTATION

At zero magnetic field, the ground state of a three-massless-flavor system at very high density is the CFL phase [3]. However, once a magnetic field is switched on, the difference between the electric charge of the u quark and that of the d and s quarks reduces the original flavor symmetry of the theory [9] and consequently also the symmetry group remaining after the diquark condensate is formed. This less-symmetric color-superconducting phase is the MCFL phase [5]. In this phase the external magnetic field modifies the structure and magnitude of the quark gap in such a way that the pairing of (rotated) electrically charged quarks is reinforced by the external field. At very strong fields -of the order of the baryon chemical potential- the pairing reinforcement is sufficient to produce a distinguishable splitting of the gap in two pieces: one that only gets contributions from pairs of neutral quarks and one that gets contributions from both pairs of neutral and pairs of charged quarks.

The CFL and MCFL phases not only differ in the structure and magnitude of the gaps. They also have different low-energy physics. The formation of the diquark condensate in the CFL case is accompanied by the appearance of nine Goldstone bosons: a singlet associated to the breaking of the baryonic symmetry $U(1)_B$, and an octet associated to the axial $SU(3)_A$ group. Four fields of the octet are charged with respect to the rotated electric charge. On the other hand, the symmetry breaking that gives rise to MCFL leaves a smaller number of Goldstone fields, all of which are neutral with respect to the rotated electric charge. This implies that no charged low-energy excitation can be produced in the MCFL phase, an effect with possible consequences for the low-energy physics of a color-superconducting star's core.

Despite the difference between MCFL and CFL, the two phases are hardly distinguishable at weak magnetic fields. The symmetry of the CFL phase can be considered as an approximated symmetry in the presence of an external magnetic field [10], as long as the field-induced mass of the charged Goldstone bosons is smaller than twice the gap, so

these mesons cannot decay into a quasiparticle-quasihole pair [8]. Since, strictly speaking, the exact symmetry in the presence of the magnetic field (ignoring quark masses) is that of MCFL, the transition from the "approximated" CFL to MCFL at some threshold field is not a phase transition, but a crossover or symmetry transmutation.

The threshold field at which CFL ceases to be a good, approximate symmetry can be found from the CFL low-energy theory of the Goldstone bosons in the presence of a weak ($k^2 \sim \tilde{e}\tilde{B} \ll \mu^2$) constant magnetic field [10]. The leading-order Lagrangian was found in [8]. It is given by

$$L = \int d^4x \frac{f_\pi^2}{4} \left\{ \left[\sum_{A=1}^{3,8} |\partial_0 \phi^A|^2 + |\partial_0 \Upsilon|^2 \right] + v_\parallel^2 \left[\sum_{A=1}^{3,8} |\partial_\parallel \phi^A|^2 + |\partial_\parallel \Upsilon|^2 \right] \right. \\ \left. + v_\perp^2 \left[\sum_{A=1}^{3,8} |\partial_\perp \phi^A|^2 + |(\partial_\perp + i\tilde{e}\tilde{A}_\perp)\Upsilon|^2 \right] \right\}, \quad (1)$$

where the fields ϕ^A denote the neutral mesons and

$$\Upsilon \equiv \begin{pmatrix} \Pi^+ \\ \kappa^+ \end{pmatrix} \quad (2)$$

represents the charged meson doublet [8].

To find the strength of the threshold field from (1), one can go to momentum space by applying the Ritus' method [11] to the case with charged scalar fields [8] in order to obtain the dispersion relation of the charged mesons

$$E^2 = \tilde{e}\tilde{B}(2n+1)v_\perp^2 + k_3^2 v_\parallel^2. \quad (3)$$

Taking into account that at zero momentum ($k_3 = 0, n = 0$) the rest energy of the charged mesons is $M_B^2 = \tilde{e}\tilde{B}v_\perp^2$, it is clear that these mesons acquire a mass due to the nonzero magnetic field [8]. These bound states will decay when their mass is equal or larger than their constituents, that is, twice the gap. Hence, the threshold field is

$$\tilde{e}\tilde{B}_{MCFL} = \frac{4}{v_\perp^2} \Delta_{CFL}^2 \simeq 12 \Delta_{CFL}^2, \quad (4)$$

where the velocity $v_\perp \approx 1/\sqrt{3}$ was approximated by its zero-field value [12]. Using $\Delta_{CFL} \sim 15 \text{ MeV}$, the order of magnitude of the field strength at which the charged mesons decouple is $\sim 10^{16} G$.

After this decoupling, the low-energy physics of the system is driven by the five neutral Goldstone bosons (including the one associated with the baryon-symmetry breaking) that characterize the MCFL phase. Therefore, the MCFL phase becomes relevant at magnetic field scales comparable to the CFL gap square, even though near the threshold field the splitting of the gaps is still negligible.

Notice the analogy between the CFL-MCFL transmutation and what can be called a "field-induced" Mott transition [8]. Mott transitions have been discussed in condensed

matter and in QCD [13, 14] to describe delocalization of bound states into their constituents at a temperature defined as the Mott temperature. By definition, the Mott temperature T_M is the temperature at which the mass of the bound state equals the mass of its constituents, so the bound state becomes a resonance at $T > T_M$. In the CFL-MCFL transmutation, the usual role of the Mott temperature is played by the threshold field \tilde{B}_{MCFL} .

MCFL-PCFL PHASE TRANSITION

The magnetic field can also influence the gluon dynamics [6, 7] because some of the gluons in the CFL have nonzero rotated charge. As shown above, at field strengths larger than \tilde{B}_{MCFL} , the system is in the MCFL phase. For fields in the range: $\tilde{e}\tilde{B}_{MCFL} \leq \tilde{e}\tilde{B} < m_M^2$, where the Meissner mass square m_M^2 of the charged gluons satisfies $\mu^2 \gg m_M^2 \gg \Delta_{CFL}^2$, the field-induced gap separation is still negligible, as corroborated by recent numerical calculations [15]. That allows one to assume that the quark gap is basically given by Δ_{CFL} , even at field strengths comparable to m_M^2 . This was the approach followed in papers [6, 7]. It led to the discovery of a new magnetic phase: the PCFL one. Let us explain how it appears.

Once $\tilde{B} \geq \tilde{B}_{PCFL} = m_M^2$, one of the modes of the charged gauge field becomes tachyonic (this is the well known "zero-mode problem" for spin-1 charged fields in the presence of a magnetic field found for Yang-Mills fields [16], for the W_μ^\pm bosons in the electroweak theory [17, 18], and even for higher-spin fields in the context of string theory [19]). Similarly to other spin-1 theories with magnetic instabilities [16]-[18], the solution of the zero-mode problem leads to the restructuring of the ground state through the formation of an inhomogeneous gauge-field condensate G , as well as an induced magnetic field due to the back reaction of the G condensate on the rotated electromagnetic field. The magnitude of the G -condensate plays the role of the order parameter for the phase transition occurring at $\tilde{B} = \tilde{B}_{PCFL}$.

Near the transition point, the amplitude of the condensate G is very small [6]. Then, the condensate solution can be found using a Ginzburg-Landau (GL) approach similar to Abrikosov's treatment of type II metal superconductivity near the critical field H_{c2} [20]. As in Abrikosov's case, the order parameter $|G|$ continuously increases from zero with the applied magnetic field, signaling a second-order phase transition towards a gluon crystalline vortex state characterized by the formation of flux tubes. Both spatial symmetries -the rotational symmetry in the plane perpendicular to the applied magnetic field and the translational symmetry- are broken by the vortex state.

Nevertheless, contrary to what occurs in conventional type-II superconductors, where the applied magnetic field only penetrates through flux tubes and with a smaller strength than that of the applied field, the gluon vortex state exhibits a paramagnetic behavior. That is, outside the flux tube the applied field \tilde{B} totally penetrates the sample, while inside the tubes the magnetic field becomes larger than \tilde{B} . This antiscreening behavior is similar to that found in the electroweak system at high magnetic field [18]. Hence, since the \tilde{Q} photons remain long-range in the presence of the condensate G , the rotated electromagnetism remains unbroken. At asymptotically large densities, because $\Delta_{CFL} \ll$

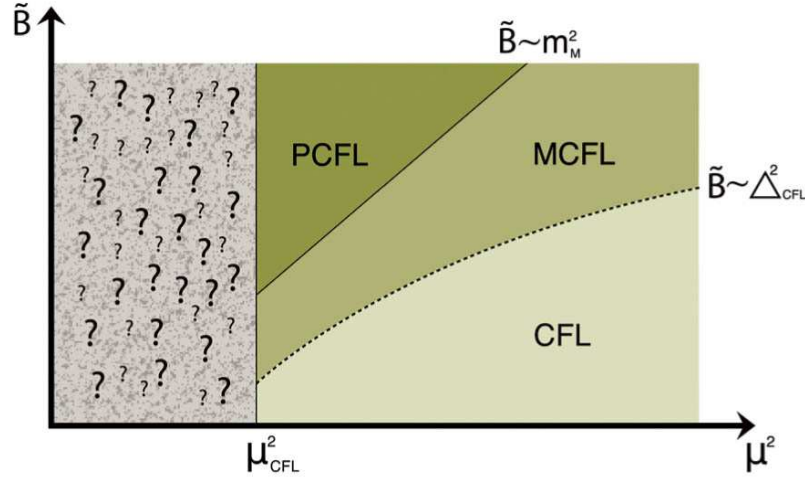


FIGURE 1. Qualitative sketch in the \tilde{B} vs μ^2 plane of the different phases of a color superconductor with three quark flavors in the presence of an external magnetic field at asymptotically high densities. The CFL phase appears here as an approximate symmetry at weak field. Thus, the line between the CFL and MCFL phases does not denote a real phase transition, but the boundary separating the approximated CFL phase from the MCFL phase. This symmetry-transmutation line is reached at field values of the order of the CFL gap square. The line between the MCFL and PCFL phases indicates a second-order phase transition curve occurring at field strengths of the order of the magnetic mass square of the charged gluons. The rectangular region to the left corresponds to moderately high densities in the presence of a magnetic field. Since the ground state at moderately high densities has not yet been investigated in the presence of a magnetic field, this region is indicated by question marks.

m_M , we have $\tilde{B}_{MCFL} \ll \tilde{B}_{PCFL}$ for each μ value.

MAGNETIC PHASES

In summary, at very high densities a three-flavor CS can be in one of three phases, depending on the magnitude of the external magnetic field. Going from low to high fields, a symmetry transmutation from CFL to MCFL will take place first, and then a second-order phase transition from MCFL to PCFL (see Fig.1). During the CFL-MCFL transmutation no symmetry breaking occurs, since in principle once a magnetic field is present, the symmetry is theoretically that of the MCFL. In practice, however, when $\tilde{B} < \tilde{B}_{MCFL} \sim \Delta_{CFL}^2$, the MCFL phase is almost indistinguishable from the CFL. Only at fields comparable to Δ_{CFL}^2 the main features of MCFL emerge through the low-energy behavior of the system. At the threshold field \tilde{B}_{MCFL} , only five of the original nine Goldstone bosons that characterize the low-energy behavior of the CFL phase remain. These are precisely the five Goldstone bosons determining the new low-energy behavior of the genuinely realized MCFL phase. Going from MCFL to PCFL is, on the other hand, a real phase transition [6, 7], as the translational symmetry, as well as the remaining rotational symmetry in the plane perpendicular to the applied magnetic field, are broken by the vortex state.

In the above considerations we have ignored the effects due to non-zero quark masses,

because we assumed a very large baryon density. However, the densities of interest for most astrophysical applications are just moderately high and for them the effects of quark masses and color and electric neutrality should not be ignored. It is plausible that if compact stars are the natural playground for color superconductivity, the magnetic phases described in this paper, or more precisely, the version of these phases at more realistic densities, may be relevant for the physics of the core of highly magnetized compact objects like magnetars. We refer the interested reader to Refs. [21] for a discussion of the inconsistencies between some observations and the standard magnetar model, and to Ref.[8] for a discussion of the possible implications of the CS magnetic phases in the solution of this conundrum.

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REFERENCES

1. S. Frautschi, in: Proc. Workshop on Hadronic Matter at Extreme Energy Density, N. Cabibbo, ed (Erice, Italy 1978); B. C. Barrois, *Nucl. Phys. B* **129** (1977) 390; D. Bailin and A. Love, *Phys. Rep.* **107** (1984) 325; M. Alford, K. Rajagopal and F. Wilczek, *Phys. Lett. B* **422** (1998) 247; R. Rapp, T. Schafer, E. V. Shuryak, and M. Velkovsky, *Phys. Rev. Lett.* **81** (1998) 53.
2. C. Thompson and R. C. Duncan, *ApJ*, **392**, L9 (1992); *Ibid.* **473**, 322 (1996).
3. M. Alford, K. Rajagopal and F. Wilczek, *Nucl. Phys. B* **537**, 443 (1999).
4. E. V. Gorbar, *Phys. Rev. D* **62**, 014007 (2000).
5. E. J. Ferrer, V. de la Incera and C. Manuel, *Phys. Rev. Lett.* **95**, 152002 (2005); *Nucl. Phys. B* **747**, 88 (2006); *PoS JHW2005*, 022 (2006); *J. Phys. A* **39**, 6349 (2006).
6. E. J. Ferrer and V. de la Incera, *Phys. Rev. Lett.* **97**, 122301 (2006).
7. E. J. Ferrer and V. de la Incera, *J. Phys. A* **40**, 6913 (2007).
8. E. J. Ferrer, V. de la Incera, nucl-th/0703034 (to appear in *Phys. Rev. D*).
9. V. A. Miransky, and I. A. Shovkovy, *Phys. Rev. D* **66**, 045006 (2002).
10. C. Manuel, *PoS JHW2005*, 011 (2006).
11. V.I. Ritus, *Ann.Phys.* 69 (1972) 555; *Sov. Phys. JETP* **48**,788 (1978) [*Zh. Eksp. Teor. Fiz.* **75**, 1560 (1978)]; E. Elizalde, E. J. Ferrer, and V. de la Incera, *Ann. of Phys.* **295**, 33 (2002); *Phys. Rev. D* **70**, 043012 (2004).
12. D. T. Son and M. A. Stephanov, *Phys. Rev. D* **61**, 074012 (2000).
13. N.F. Mott, *Rev. Mod. Phys.* **40**, 677 (1968); *Metal-Insulator Transitions* (Taylor and Francis, London, 1974).
14. J. Hüfner, S.P. Klevansky, and P. Rehberg, *Nucl. Phys. A* **606**, 260 (1996).
15. Fukushima and H. J. Warringa, arXiv:0707.3785 [hep-ph]; Jorge L. Noronha, Igor A. Shovkovy, arXiv:0708.0307 [hep-ph].
16. V. V. Skalozub, *Sov. J. Nucl. Phys.* **28**, 113 (1978); N. K. Nielsen and P. Olesen, *Nucl. Phys. B* **144**, 376 (1978).
17. V. V. Skalozub, *Sov. J. Nucl. Phys.* **43**, 665 (1986); **45**, 1058 (1987).
18. J. Ambjorn and P. Olesen, *Nucl. Phys. B* **315**, 606 (1989); *Phys. Lett. B* **218**, 67 (1989).
19. S. Ferrara and M. Porrati, *Mod. Phys. Lett. A* **8**, 2497 (1993); E. J. Ferrer and V. de la Incera, *Int. Jour. of Mod. Phys. A* **11**, 3875 (1996).
20. A. A. Abrikosov, *Sov. Phys. JETP* **5**, 1174 (1957).
21. J. Vink and L. Kuiper, *Mon. Not. Roy. Astron. Soc. Lett.* **370**, L14 (2006); R-X Xu, astro-ph/0611608.